A thermodynamic study of a magnetized neutral vector boson gas at finite temperature

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We study the thermodynamic properties of a relativistic magnetized neutral vector boson gas at any temperature. We compare the results with the low temperature (LT) and the non relativistic (NR) descriptions of this gas.[b]

Motivation

We would like to discuss our contribution to the modelling of matter inside Neutron Stars (NS). So far there has been no consensus with respect to the nature of the matter that makes up the nucleus of these stars. Nevertheless, it has been proposed that at some stage of their evolution, NS might contain certain amount of bosons formed by the pairing of two neutrons. The thermodynamic study of boson gases is commonly done in the low T limit or at $T = 0$. That is why we set out to carry out the study without neglecting the effects of temperature.

Neutral vector boson gas. Antiparticles

The gas we are analysing is composed by non-interacting bosons, with **spin-1**, mass **m**, zero electric charge, magnetic moment **k**, and under the action of a constant and uniform magnetic field $\vec{B} = (0, 0, B)$. Bosons have three possible projections of the spin in the z direction $S_z = -1, 0, 1$.

The thermodynamic study was made through the thermodynamic potential $\Omega(\mu, T, B) = \Omega_{st}(\mu, T, B) + \Omega_{vac}(B)$, described in [1], which has been separated into two terms, the statistic and the vacuum contributions. This potential was obtained without using any type of approximation, so it is valid for any temperatures.

The LT limit is obtained by assuming $T \ll m$ and neglecting the antiparticles contribution, as well as the contribution coming from the spin states with $s = 0, -1$. This last assumption is equivalent to considering a strong field limit, where the transitions of bosons from the $s = 1$ state to any excited spin state will be forbidden.

In the NR limit $p_3, p_\perp, \kappa B \ll m$. These approximations are equivalent to neglecting the vacuum and the antiparticles contributions.

The difference between particles and antiparticles $(\rho^+ - \rho^-)$ is a conserved magnitude that we call $\rho_{st} = \frac{\partial\Omega(\mu,T,B)}{\partial\mu}$. However, when the condensed state is reached ρ_{st} is no longer conserved. Instead, the magnitude that remains constant is $\rho_{gs} + \rho_{st}$, where the first term represents the particle density on the ground

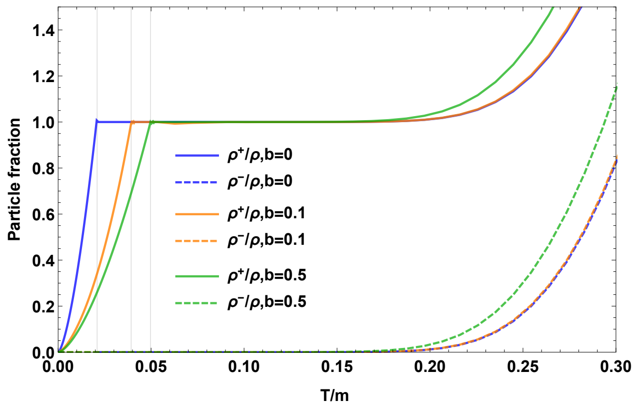

Figure 1: Particle (solid lines) and antiparticle (dashed lines) fraction as function of temperature for several values of the magnetic field.

state. From Figure 1 we see that the antiparticle density begins to be appreciable for temperatures such that $T \sim m/5$. Therefore, in this temperature range the presence of antiparticles may have an important influence on the thermodynamic properties of the gas.

Bose-Einstein condensation

Bose-Einstein condensation (BEC) of magnetized Bose gases, not only depends on T and ρ, but it also depends on the magnetic field. As it is shown in Fig. 2, the

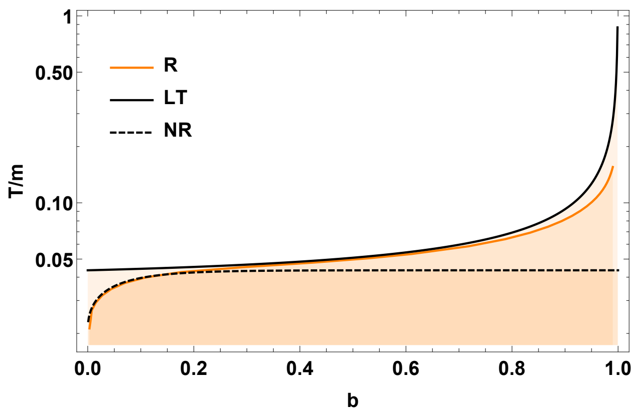

Figure 2: BEC phase diagram in the T vs B plane. The white region corresponds to the free gas, and the colored one to the condensate.

increasing of b ("b" is the dimensionless magnitude of the magnetic field $b = B/B_c$, where $B_c = m/2k$ is the critical magnetic field) augments T_c. As $b \to 1$ ($B \to B_c$) the critical temperature of the relativistic gases diverge, while in the NR limit it approaches a constant value.The saturation of $T_c^{NR}(b)$ is caused by the absence of a critical magnetic field. It is also worth noting in Fig. 2 that at $b = 0$, $T_c^R(0) = T_c^{NR}(0) \neq T_c^{LT}(0)$. This difference between R, NR and LT arises because in the LT limit, the spin states with $s = 0, -1$ were neglected and all the particles were considered to have $s = 1$.

Magnetic Properties

The magnetization of this boson gas consists of three contributions: ground state, statistics and the vacuum, as it is shown in Fig. 3. From the graph we can see that

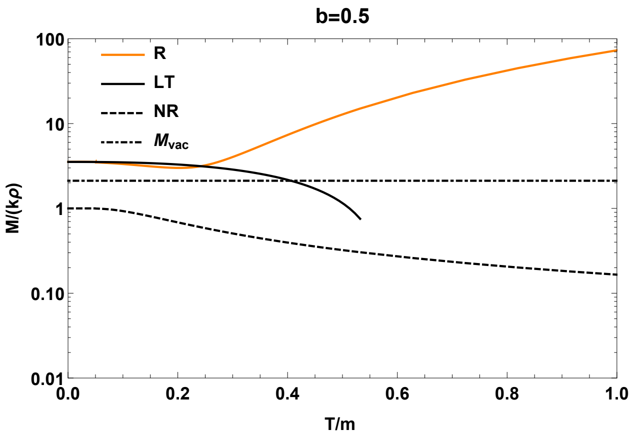

Figure 3: The magnetization as a function of temperature.

the $\mathcal{M}_{vac}$ is higher than the maximum of $\mathcal{M}^{NR}$ and comparable to $\mathcal{M}_{st}$. As a consequence, the magnetization of the relativistic case differs from $\mathcal{M}^{NR}$ at $T = 0$. As the temperature increases, the NR magnetization decreases and goes to zero for $T \to \infty$ [2]. The magnetization of the relativistic gas shows the same behavior as the non-relativistic limit up to $T \sim 0.2\, m$, while after this value $\mathcal{M}(\mu, T, b)$ it begins to grow and it increases in several orders of magnitude.The LT magnetization also decreases when T increases, but its behavior is quite different from the other two cases, becoming negative around $T \sim 0.5\ m$ (the point where the curve ends). This negative magnetization does not imply that the gas is displaying a diamagnetic behavior; it is again a consequence of neglecting the states with $s = 0$ and $s = -1$ in the LT limit.

An interesting property is obtained when the limit $b \to 0$ in the magnetization is applied. A spin-one BEC that was under the action of an external magnetic field, will remain magnetized even if the external magnetic field is somehow "disconnected". This phenomenon is known as Bose-Einstein ferromagnetism, although it is not caused by a spin-spin interaction, it is rather a consequence of BEC since all the bosons in the ground state have $s = 1$.

Conclusions

The numerical study of the magnetized neutral vector boson gas in astrophysical conditions allowed us to evaluate the relative influence of the particle density, the magnetic field and temperature on the system. These calculations were carried out for a boson with mass $2m_n$ and magnetic moment $2k_n$, where m_n and k_n are the mass and magnetic moment of the neutron and $\rho = 1.30 \times 10^{39}\mathrm{cm}^{-3}$. However, in all cases the presentation and discussion of the results have been done in a general framework and the conclusions are in this way valid beyond the chosen particle.

Depending on T there are two distinct regimes for the behaviour of the gas: $T \ll m$ where the effects of the magnetic field dominate the system and, $T \gg m$ where the temperature effects dominate, being the most important of these effects, the existence of a non-negligible fraction of antiparticles.

The comparison of all temperature relativistic calculations with their corresponding NR and LT limits allowed us to establish the validity ranges of these approximations as well as the physics they ignore. Something similar happens with the vacuum pressure and with the magnetization: they are usually neglected. However, their effects are important when considering strong magnetic fields.

Notes

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b. Original version of this article is Ref. [2, 3, 4]

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